MASTER'S RESEARCH PROPOSAL

Learning energy functionals
via Physics Informed Neural Networks
for the stability analysis of weakly-hyperbolic systems

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Key words: linear systems, operator theory, stability Lyapunov analysis, neural networks, water treatment.

Expected skills: Candidates should have a strong background in applied mathematics and informatics, a knowledge of neural networks algorithms, and basic skills in control theory.

Research aim

The main objective is to develop a fast, safe, and scalable computational tool for predicting the energy and stability of weakly-hyperbolic systems. Using physics informed neural netwroks, we aim to approximate Lyapunov operators that describe the system's stability properties. This approach bypasses the need to solve computationally expensive **infinite-dimensional Lyapunov** equations while guaranteeing predictive accuracy. Reliable stability prediction can optimize the computation of the Lyapunov operator kernel, reduce energy consumption, and prevent operational failures.
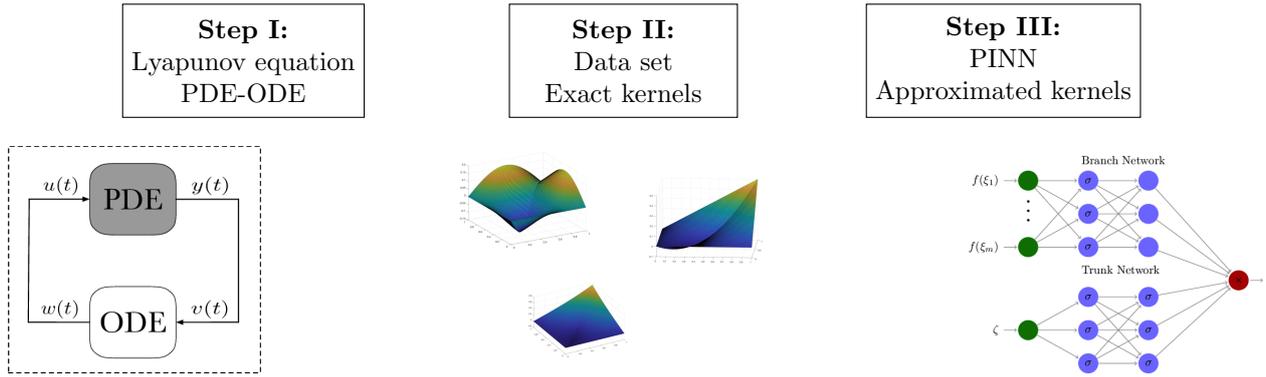
This tool will be applied to **water treatment plants**, where water flows are described by weakly-hyperbolic transport partial differential equations, with complex boundary conditions. Several simplified models for this kind of systems can be obtained. The computation of Lyapunov functionals is then crucial to deduce stability behavior of its solutions.

The project also opens the possibility for candidates to continue in a PhD program, focusing on other partial differential equations or AI-driven control optimization.

Context and issues

Many industrial systems, such as water purification units, are governed by partial differential equations (PDEs) describing transport phenomena. When these physical processes are coupled with digital controllers, classical energy-based methods cannot fully capture the system's dynamics. Lyapunov functionals offer a mathematical substitute for energy, providing a measure of system stability. However, computing the Lyapunov operator explicitly is often infeasible and computationally expensive.

This project addresses the question: **how can we efficiently predict stability for complex transport partial differential equations?**

Step I:	Step II:	Step III:
Lyapunov equation	Data set	PINN
PDE-ODE	Exact kernels	Approximated kernels

Work plan

- **Data generation**: Compute exact Lyapunov operators for a discretized grid of parameter values (transport velocities, boundary parameters, control gains).

- **Neural network training**: Train a PINN to learn the mapping from system parameters to Lyapunov operators.

- **Generalization and validation**: Use the trained network to predict operators for new parameter sets, with guaranteed error bounds.

- **Application to water treatment systems**: Implement the method in Python or MATLAB for **transport PDE** coupled with a water treatment plant controller.

- **Comparison and analysis** (eventually): Benchmark against classical numerical methods (finite element, polynomial projection) in terms of computation time and accuracy.

The results could be published in computer science journals and presented at international conferences (IEEE CDC, IFAC WC).

References

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