

Stability and Safety Guarantees for Learning-based Control of Hybrid Dynamical Systems

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Target Profiles

M2 student with good academic results, solid background in **control theory**, **machine learning** and **mathematics**, and excellent writing and communication in English.

Experience in hybrid dynamic systems, machine learning, and programming are appreciated.
CV and grades (including rankings if available) must be sent to: moussa.labbadi@enib.fr

Research Environment: The internship will be conducted at iRDL, Plouzané.

Context and scientific scope

Neural networks offer the flexibility needed to control complex unknown dynamical systems, but their nonlinear behavior makes safety and stability difficult to guarantee. These challenges are particularly relevant to hybrid dynamical systems, in which continuous flows interact with switching, discrete controller updates, and state resets. This internship will investigate how the expressive power of recurrent neural networks (RNNs) can improve control performance while retaining rigorous closed-loop guarantees on a specified set domain.

You will develop methods for analysis, verification, and controller design that incorporate certifiable properties into the architecture and learning process. Physical insight, prior system knowledge, and stabilizing baseline controllers will guide initialization and certified fallback strategies. The central approach combines hybrid Lyapunov theory [1], neural controller synthesis [2], and homogeneous approximations [3].

The initial class comprises nonlinear hybrid plants with state-dependent switching or resets. The augmented state z includes the plant state and the RNN memory, with mode or timer states included when required. The closed-loop dynamics are represented as

$$z \in C : \quad \dot{z} \in F_\theta(z), \quad z \in D : \quad z^+ \in G_\theta(z),$$

where C, D are the flow and jump sets, F_θ, G_θ incorporate the controller, and θ denotes its parameters. The target set $\mathcal{A}$ must be compatible with both regimes; in particular, admissible jumps must preserve it.

A specific Lyapunov construction will combine a homogeneous candidate near the target with a neural candidate on an outer annular region:

$$V(z) = \chi(z)V_H(z) + (1 - \chi(z))V_\psi(z), \quad V_\psi(z) = \|T_\psi(z)\|^2.$$

Here $0 \leq \chi \leq 1$ is a smooth transition function, equal to one in the inner region and zero outside the outer region. The neural feature map T_ψ , the homogeneous candidate, and their combination require verification; a squared-norm parameterization alone does not establish positive definiteness. For a differentiable candidate, the analysis will seek comparison bounds and hybrid decrease conditions of the form

$$\begin{aligned} \alpha_1(|z|_{\mathcal{A}}) &\leq V(z) \leq \alpha_2(|z|_{\mathcal{A}}), \\ \langle \nabla V(z), f \rangle &\leq -W_C(z), \quad z \in C, \quad f \in F_\theta(z), \\ V(g) - V(z) &\leq -W_D(z), \quad z \in D, \quad g \in G_\theta(z). \end{aligned}$$

The comparison functions are of class $\mathcal{K}_\infty$, and W_C, W_D are nonnegative. Attractivity requires appropriate strict decrease or persistence assumptions, together with solution continuation. The flow calculation must include derivatives of χ , and the jump calculation must use the full candidate at both the pre-jump and post-jump states.

Scientific objectives and work plan

1. **State of the art in learning-based stability and stabilization of hybrid systems.** Review neural Lyapunov functions, RNN controllers, verification methods, and hybrid barrier certificates. Include learner–verifier methods for discrete-time hybrid systems admitting mixed-integer representations [4], and distinguish their assumptions from those of flow-and-jump models. Review Zubov’s method and neural PDE formulations for regions of attraction [5,6]. Their extension to hybrid dynamics must also account for jumps.
2. **RNN-based stability analysis and stabilization.** Develop controllers and neural Lyapunov candidates for the augmented closed-loop system. Incorporate positivity and flow/jump conditions into learning, and investigate architectures with intrinsic stability or robustness properties. Explore quadratic constraints and semidefinite programming where the plant and activation structure permit them. State regularity, persistence, and dwell-time assumptions and verify inequalities throughout the claimed domain.
3. **RNN-based stability and stabilization using homogeneous approximations.** Extend the construction in [3] to homogeneous approximations of the augmented flow and reset dynamics. Check compatibility with weighted dilations and bound approximation remainders. Establish sufficient conditions for combining the homogeneous and neural candidates, including transitions and reset images, and compare the resulting certificates with Objective 2. Approximation at infinity is a possible extension after local results are validated.
4. **Safety of the designed controllers.** Define a safe set $\mathcal{S} = \{z : b(z) \geq 0\}$ containing $\mathcal{A}$. Investigate control barrier functions [7] and hybrid invariance conditions [8]. Use mode-specific local CBFs to study safe switching [9], explicitly checking reset and RNN-memory updates. Under suitable regularity and feasibility assumptions, flows must preserve $\mathcal{S}$, while jumps must satisfy $G_\theta(D \cap \mathcal{S}) \subseteq \mathcal{S}$. Study compatibility with Lyapunov constraints, actuator limits, and certified fallback control; quantify robustness margins where supported by the model.

The indicative six-month schedule is:

Month 1	Literature review, benchmark selection, and closed-loop hybrid modelling.
Month 2	RNN controller and neural Lyapunov construction with initial verification.
Months 3–4	Homogeneous approximations, combined certificates, and analytical results.
Month 5	Safety conditions, barrier certificates, and integration with stabilization.
Month 6	Benchmark evaluation, reproducible code, and final report.

Evaluation will start with low-dimensional nonlinear reset systems and, where feasible, progress to a more realistic hybrid-control benchmark. The final report will distinguish proved guarantees from empirical evidence and identify the assumptions and domain associated with each result.

References

- [1] Rafal Goebel, Ricardo G. Sanfelice, and Andrew R. Teel. *Hybrid Dynamical Systems: Modeling, Stability, and Robustness*. Princeton University Press, Princeton, NJ, 2012.
- [2] Fangda Gu, He Yin, Laurent El Ghaoui, Murat Arcak, Peter Seiler, and Ming Jin. Recurrent Neural Network Controllers Synthesis with Stability Guarantees for Partially Observed Systems. *Proceedings of the AAAI Conference on Artificial Intelligence*, 36(5):5385–5394, 2022.
- [3] Artemii Zenkin, Denis Efimov, Rosane Ushirobira, and Alexey Bobtsov. A neural network-based stability analysis and stabilization through homogeneous approximations. Manuscript, HAL: hal-05718446, 2026.
- [4] Shaoru Chen, Mahyar Fazlyab, Manfred Morari, George J. Pappas, and Victor M. Preciado. Learning Lyapunov functions for hybrid systems. arXiv preprint arXiv:2012.12015v1, December 2020. Version 1, 22 December 2020.
- [5] Vladimir I. Zubov. *Methods of A. M. Lyapunov and Their Application*. P. Noordhoff, Groningen, 1964. English edition edited by Leo F. Boron.
- [6] Jun Liu, Yiming Meng, Maxwell Fitzsimmons, and Ruikun Zhou. Physics-informed neural network Lyapunov functions: PDE characterization, learning, and verification. *Automatica*, 175:112193, 2025.
- [7] Aaron D. Ames, Xiangru Xu, Jessy W. Grizzle, and Paulo Tabuada. Control barrier function based quadratic programs for safety critical systems. *IEEE Transactions on Automatic Control*, 62(8):3861–3876, 2017.

- [8] Mohamed Maghenem and Ricardo G. Sanfelice. Sufficient conditions for forward invariance and contractivity in hybrid inclusions using barrier functions. *Automatica*, 124:109328, 2021.
- [9] Shuo Yang, Mitchell Black, Georgios Fainekos, Bardh Hoxha, Hideki Okamoto, and Rahul Mangharam. Safe control synthesis for hybrid systems through local control barrier functions. In *2024 American Control Conference (ACC)*, 2024.